Washington University in St. Louis

Clutter Scattering Function Estimation and Moving Target Estimation from Multiple STAP Datacubes

Daniel R. Fuhrmann, Lisandro Boggio, John Maschmeyer
Joseph A. O'Sullivan, and Roger Chamberlain

Electronic Systems and Signals Research Laboratory
Department of Electrical and Systems Engineering
Department of Computer Science and Engineering
Washington University in St. Louis
St. Louis, Missouri 63130

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OUTLINE

1. Introduction
2. Problem Formulation
3. Cramér-Rao Bound
4. EM Algorithm
5. Simulation Results
6. Moving Target Detection
7. Full-Scale Simulation
8. Conclusions

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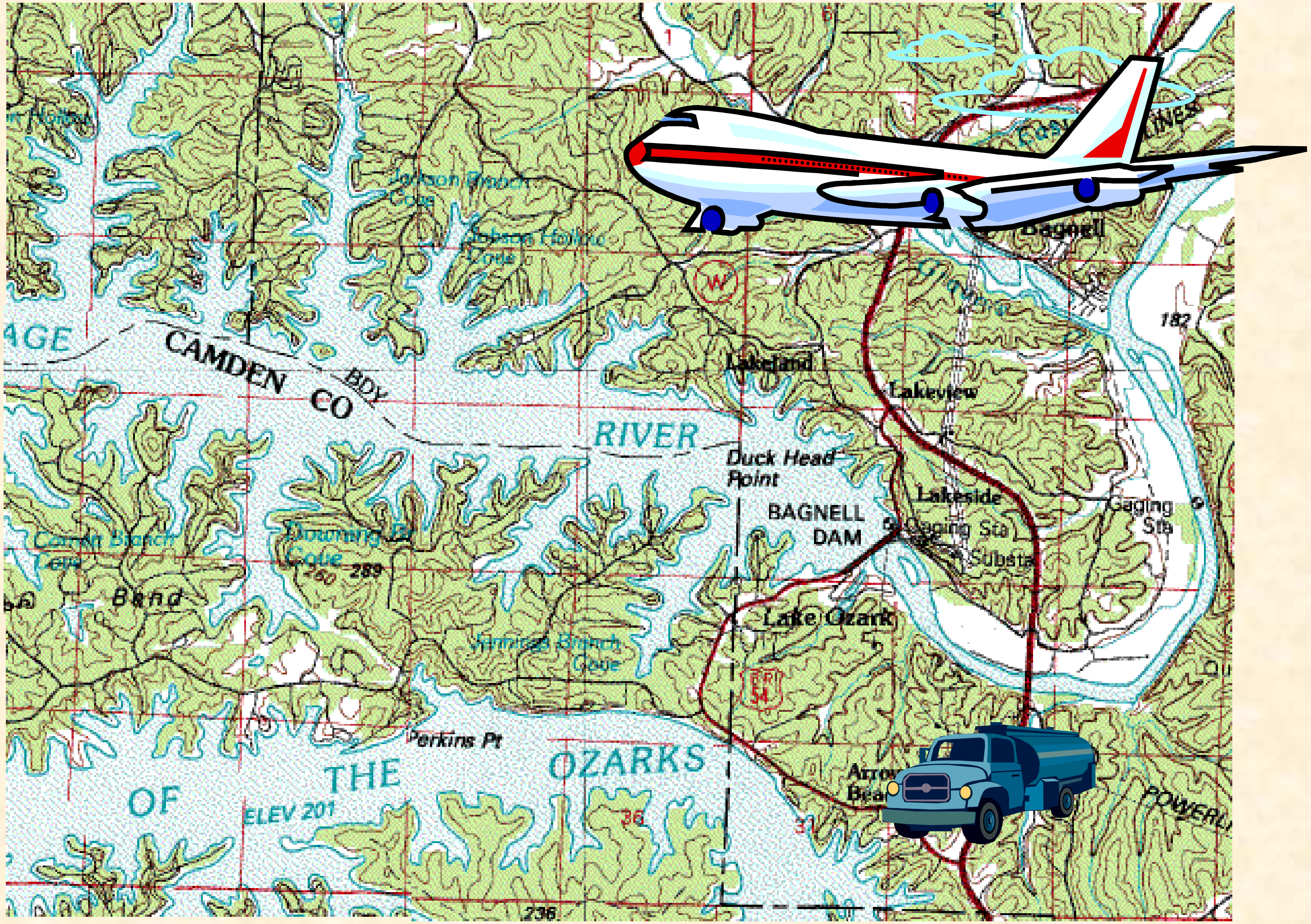
CONTEXT


CAMDEN CO
BDY
RIVER
Duck Head Point
BAGNELL DAM
Lakeland
Lakeview
Lakeside
Bagnell
Gaging Sta
Lake Ozark
Perkins Pt
THE
OZARKS
OF
ELEV 201
Bend
182
236

CONTEXT

- Airborne multisensor pulse-Doppler surveillance radar
- Arbitrary flight path around region of interest
- Ground subdivided into pixels or ground patches
- Known range, and angle of each patch with respect to platform
- Known illumination pattern

Objective: Determine ground scattering function and detect moving targets

APPROACH

- Data modeling: structured covariance
  - Received data modeled as *0*-mean complex Gaussian vectors whose covariances are linear transformations of the scattering function

- Maximum-likelihood methodology is used to estimate the unknown scattering function
  - Expectation-Maximization (EM) algorithm used to compute maximum-likelihood estimate

RELATIONSHIP TO OTHER WORK

- Method of using noncoherent datacubes to estimate clutter scattering function proposed by AlphaTech group in the 2003 KASSPER Workshop. They use least-squares estimation of complex reflectivity, whereas we propose maximum-likelihood estimation of clutter scattering function. We also include explicit illumination term in data model.

- Structured covariance EM algorithm extends work of ourselves and others, including Moulin, Robey, Barton, Lanterman, and Rieken.

OUTLINE

PROBLEM FORMULATION

- Region pixelized into *N* ground patches
  - Size of patch commensurate with radar's resolution

- Pulse waveform transmitted at instances $k=1, 2, \ldots, K$, with known illumination pattern

- Received data across the *M* sensors and *L* range gates:

$$\mathbf{z}_k = \left[z_k(1) \quad z_k(2) \quad \ldots \quad z_k(LM) \right]^T \sim CN(\,\mathbf{0}\,, \mathbf{R}_k\,)$$

where

$$\mathbf{R}_k = \sum_{n=1}^{N} \underbrace{\mathbf{a}(n,k)}_{\text{Response vector for the } n^{th} \text{ patch at } k^{th} \text{ look}} \mathbf{a}^H(n,k) \underbrace{\sigma_n}_{\text{Value of the scattering function for the } n^{th} \text{ patch}} \underbrace{\lambda_{nk}}_{\text{Incident energy on the } n^{th} \text{ patch at } k^{th} \text{ look}}$$

COMPACT NOTATION

- $\mathbf{R}_k$ is block diagonal; each block corresponds to one range gate

- Compact notation $$\mathbf{R}_k = \mathbf{A}_k \left(\boldsymbol{\Sigma} \boldsymbol{\Lambda}_k \right) \mathbf{A}_k^H$$

where $$\mathbf{A_k} = \left[a(1,k) \cdots a(N,k) \right]$$

$$\boldsymbol{\Lambda}_k = \begin{bmatrix} \lambda_{1k} & & & \mathbf{0} \\ & \lambda_{2k} & & \\ & & \ddots & \\ \mathbf{0} & & & \lambda_{Nk} \end{bmatrix} \qquad \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1 & & & \mathbf{0} \\ & \sigma_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \sigma_N \end{bmatrix}$$

Objective: Maximum Likelihood estimate of Σ

MAXIMUM LIKELIHOOD ESTIMATION

- Model of the received data

$$f(\mathbf{z}_1,\mathbf{z}_2,\cdots\mathbf{z}_K)=\prod_{k=1}^{K}\pi^{-M}\det(\mathbf{R}_k)^{-1}e^{-\mathbf{z}_k^H\mathbf{R}_k^{-1}\mathbf{z}_k}$$

- Goal:

$$\hat{\boldsymbol{\Sigma}}(\mathbf{z}_1,\mathbf{z}_2,\cdots,\mathbf{z}_K)=\max_{\boldsymbol{\Sigma}\text{ s.t. }\boldsymbol{\Sigma}\in S}\left\{L\overset{\Delta}{=}\sum_{k=1}^{K}-\log(\det(\mathbf{R}_k))-z_k^H\mathbf{R}_k^{-1}z_k\right\}$$

where $$S=\{\boldsymbol{\Sigma}:\sigma_n\geq 0\ \forall\ n=1,2,\cdots,N\}$$

$$\hat{\boldsymbol{\Sigma}}(\mathbf{z}_1,\mathbf{z}_2,\cdots,\mathbf{z}_K)=\begin{bmatrix}\hat{\sigma}_1 & & & \mathbf{0}\\ & \hat{\sigma}_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \hat{\sigma}_N\end{bmatrix}$$

Nonlinear optimization problem

OUTLINE

FISHER INFORMATION

- Fisher information matrix – element (i, j)

$$J_{ij} = \sum_{k=1}^{K} tr\left(\mathbf{R}_k^{-1} \frac{\partial \mathbf{R}_k}{\partial \sigma_i} \mathbf{R}_k^{-1} \frac{\partial \mathbf{R}_k}{\partial \sigma_j} \right)$$

where

$$\frac{\partial \mathbf{R}_k}{\partial \sigma_i} = a(i,k)\, a^H(i,k)\, \lambda_{ik}$$

Manipulating matrix traces:

$$J_{ij} = \sum_{k=1}^{K} \lambda_{ik}\, \lambda_{jk} \left| a^H(i,k) \mathbf{R}_k^{-1} a(j,k) \right|^2$$

CRAMER – RAO BOUND

- Cramér – Rao bound for an unbiased estimator

$$E_{\mathbf{z}_1,\mathbf{z}_2,\cdots,\mathbf{z}_K}\left\{\begin{bmatrix}\hat{\sigma}_1\\ \hat{\sigma}_2\\ \vdots\\ \hat{\sigma}_N\end{bmatrix}\begin{bmatrix}\hat{\sigma}_1 & \hat{\sigma}_2 & \cdots & \hat{\sigma}_N\end{bmatrix}\right\} \geq \mathbf{J}^{-1}$$

OUTLINE

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EM ALGORITHM

Unknown parameters | Complete data | Incomplete data

$$\Sigma \longrightarrow \boxed{P(\mathbf{u}_k \mid \Sigma)} \xrightarrow{\mathbf{u}_k} \boxed{P(\mathbf{z}_k \mid \mathbf{u}_k)} \longrightarrow \mathbf{z}_k$$

Many-to-one mapping

$$\mathbf{u}_k \sim CN(0, \mathbf{\Lambda}_k \mathbf{\Sigma})$$

$$\mathbf{z}_k = \mathbf{A}_k \mathbf{u}_k$$

- Complete-data sufficient statistic: $\dfrac{1}{K}\sum_{k=1}^{K} \dfrac{\left|u_{n,k}\right|^2}{\lambda_{nk}}$

EM ALGORITHM

- At each iteration, compute conditional expectation of complete-data sufficient statistics:

$$\hat{\sigma}_n^{(p+1)} = E\left[\frac{1}{K}\sum_{k=1}^{K} \frac{\left|u_{n,k}\right|^2}{\lambda_{nk}} \,\middle|\, \mathbf{z}_k, \hat{\Sigma}^{(p)} \right]$$

$$\hat{\boldsymbol{\Sigma}}^{(p+1)} = \hat{\boldsymbol{\Sigma}}^{(p)} + \frac{1}{K}\sum_{k=1}^{K} \boldsymbol{\Lambda}_k diag\left[\hat{\boldsymbol{\Sigma}}^{(p)} \mathbf{A}_k^H \left(\mathbf{R}_k^{-1}(p)\mathbf{z}_k\, \mathbf{z}_k^H \mathbf{R}_k^{-1}(p) - \mathbf{R}_k^{-1}(p)\right) \mathbf{A}_k \hat{\boldsymbol{\Sigma}}^{(p)}\right]$$

$$\mathbf{R}_k(p) = \mathbf{A}_k \boldsymbol{\Lambda}_k \hat{\boldsymbol{\Sigma}}^{(p)} \mathbf{A}_k^H$$

OUTLINE

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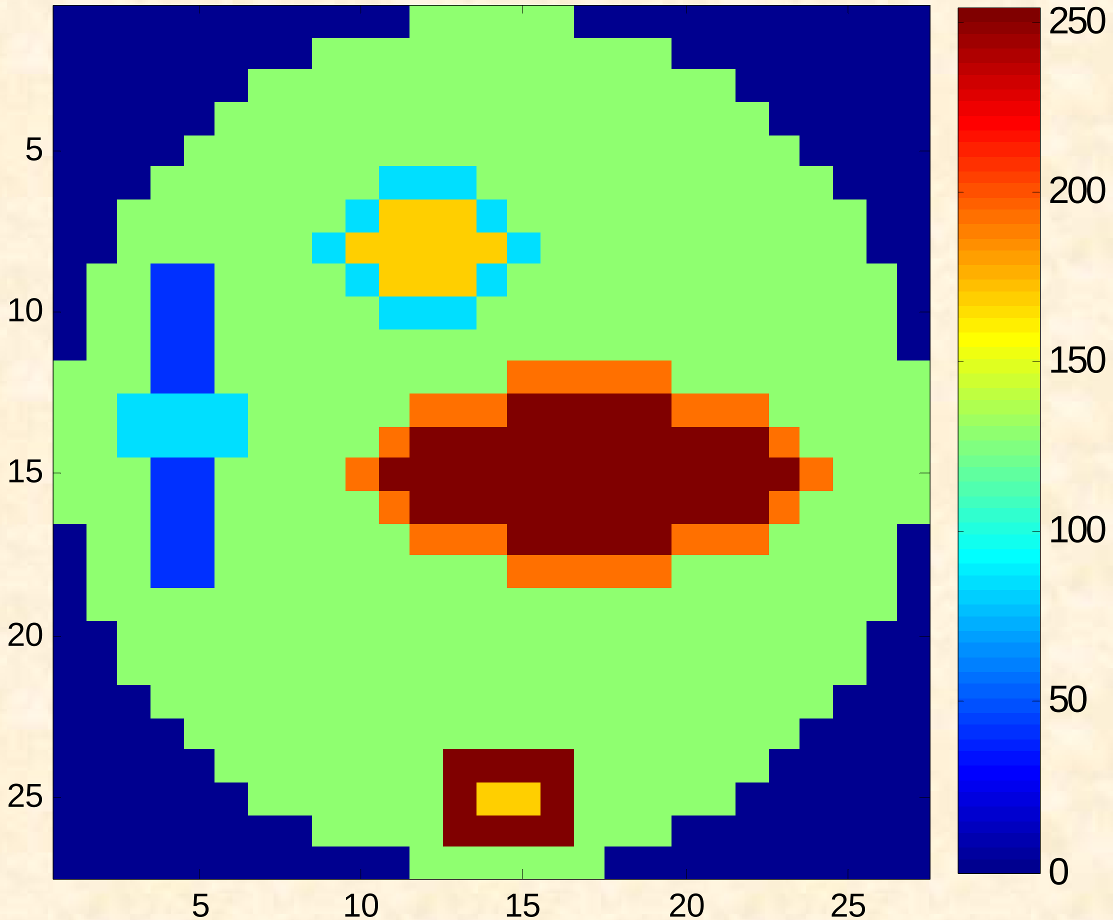
SIMULATION RESULTS



Artificial scattering function

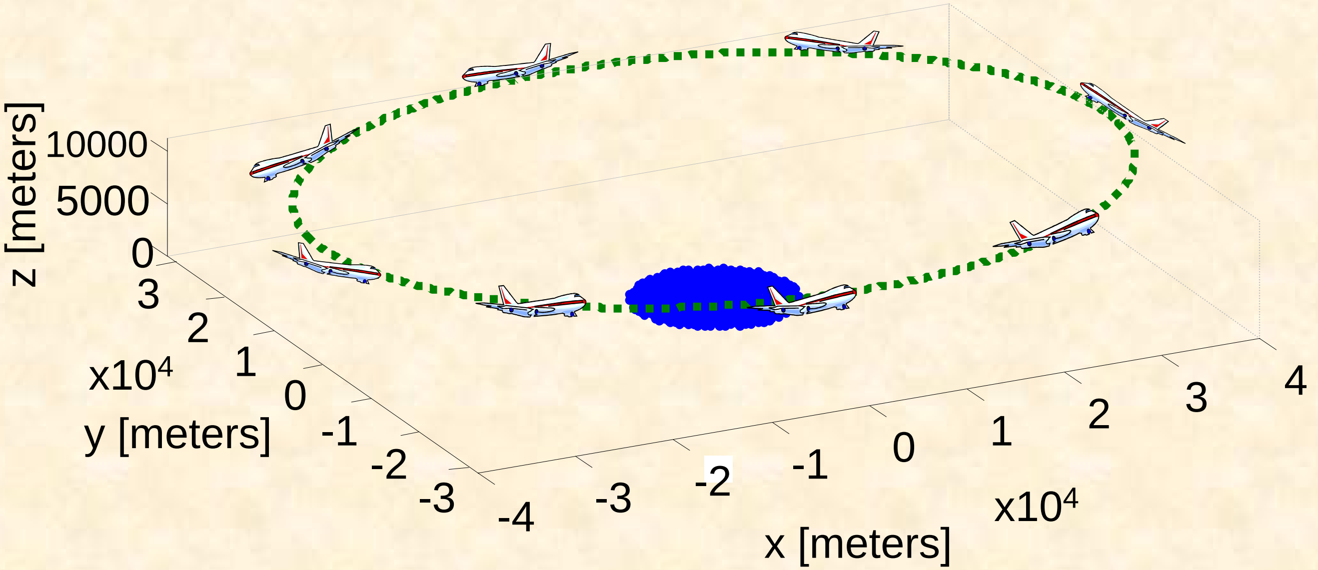
Region of interest

Circular area
16km diameter
554 pixels
(~600m on side)

Antenna

8-element
uniform linear
array

SIMULATION RESULTS

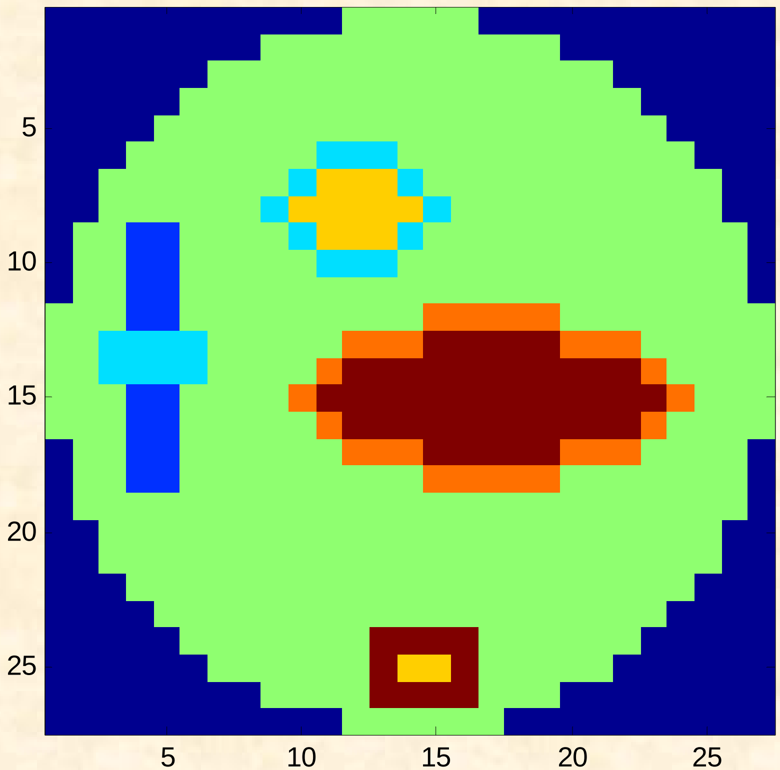


Platform

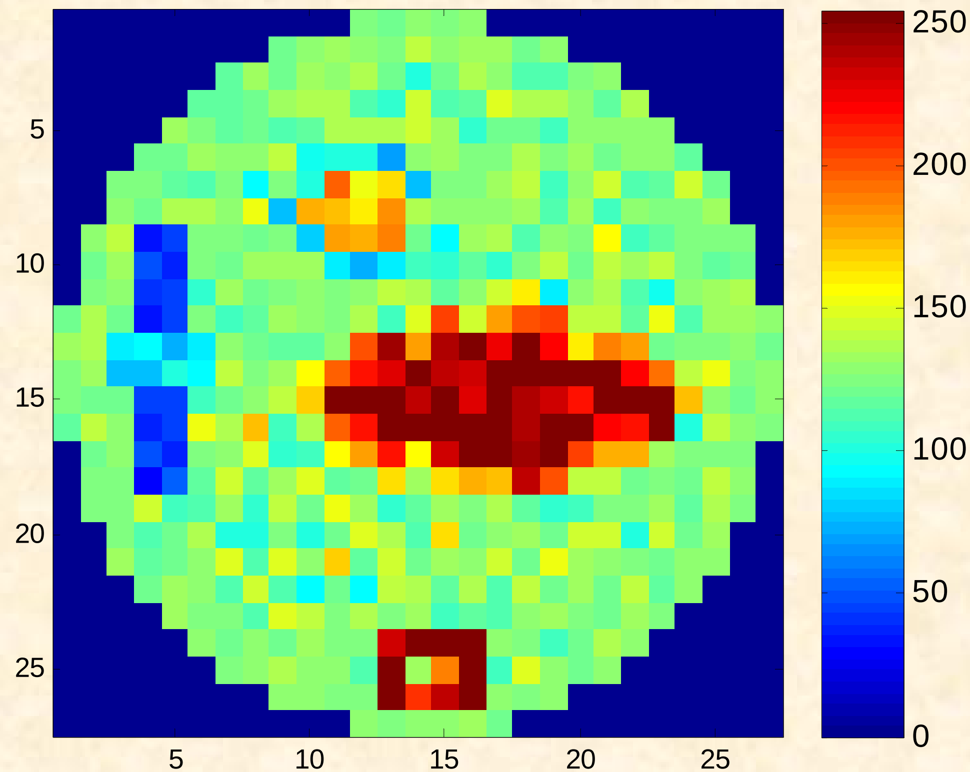
- Altitude: 11km
- Elliptical flight centered at region of interest (40km – 32km)
- 8 different viewpoints

SIMULATION RESULTS

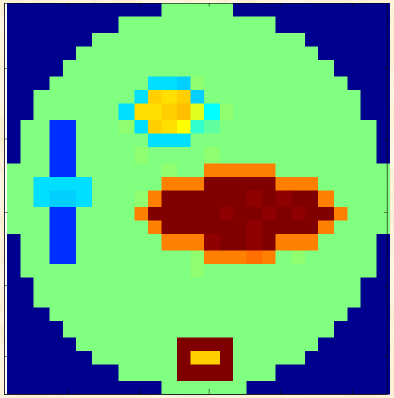
True Scattering Function



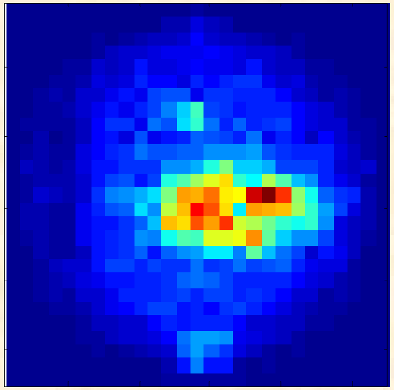
Estimated Scattering Function



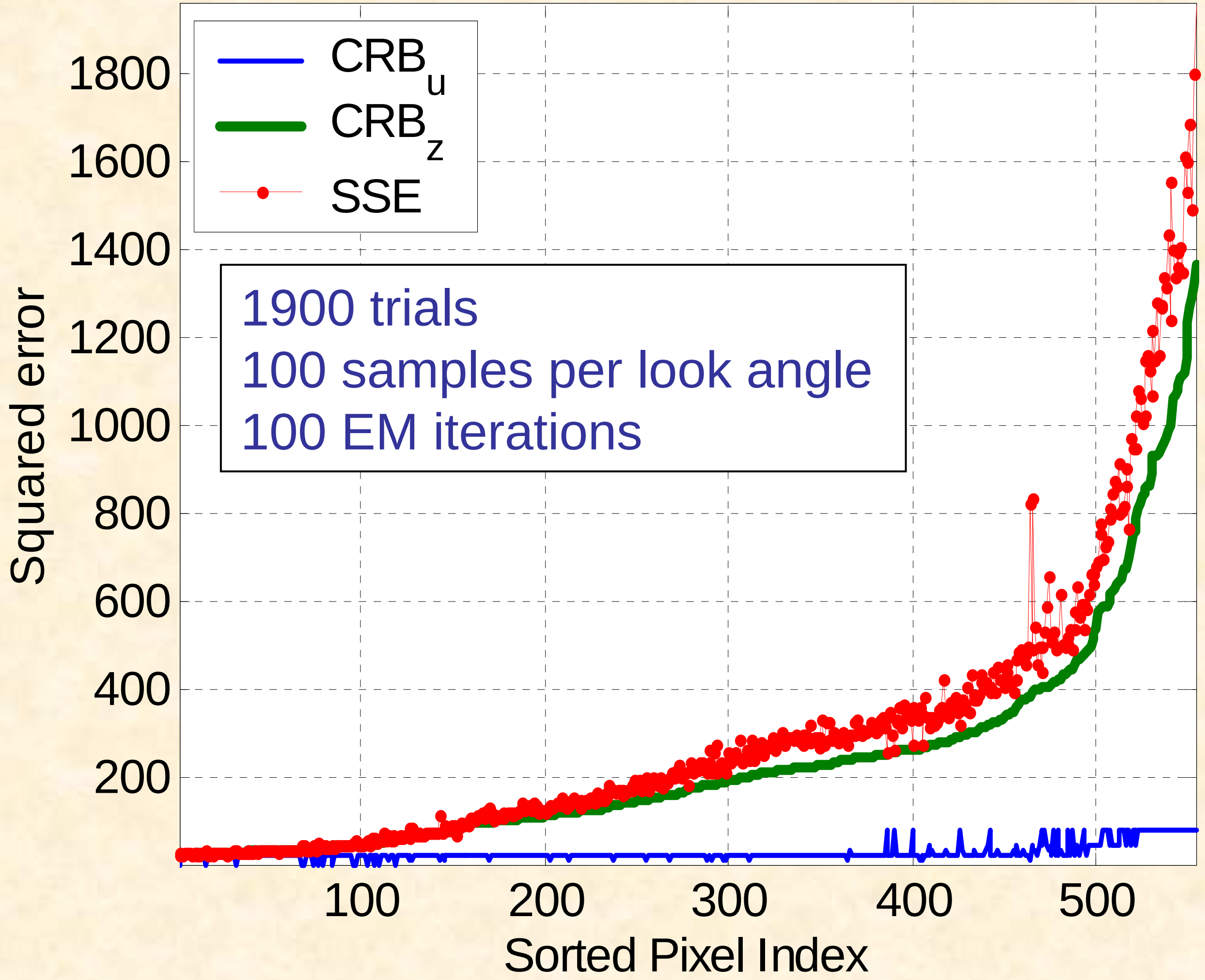
CRAMER-RAO BOUND (sorted in ascending order of CRB)



Sample mean



Squared error



INCORPORATING LAND USE DATA

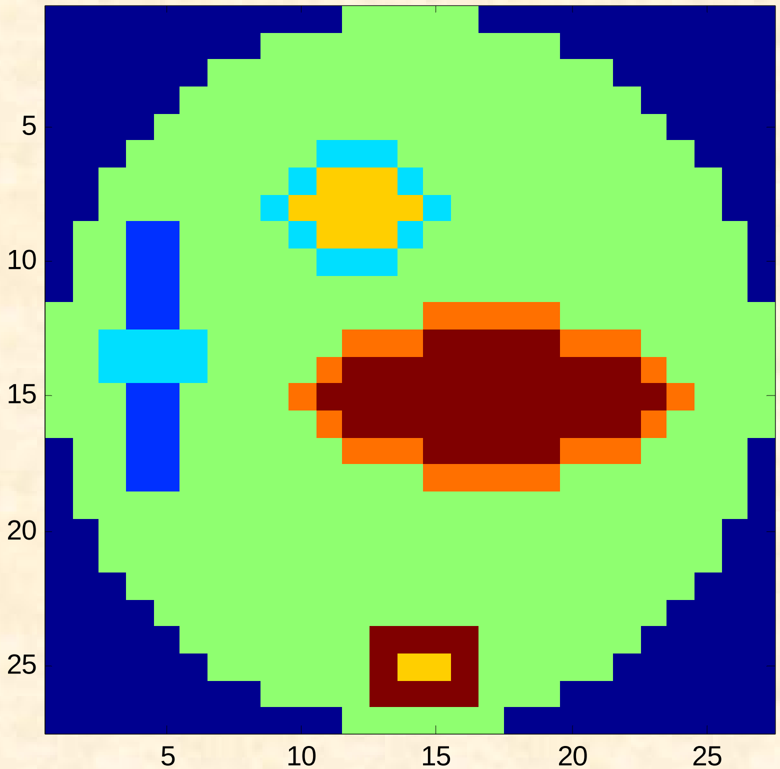
- In many geographical information systems, ground patches are labeled with land-use values

- Assume all pixels with the same land-use label *L* have the same scattering function σ_L

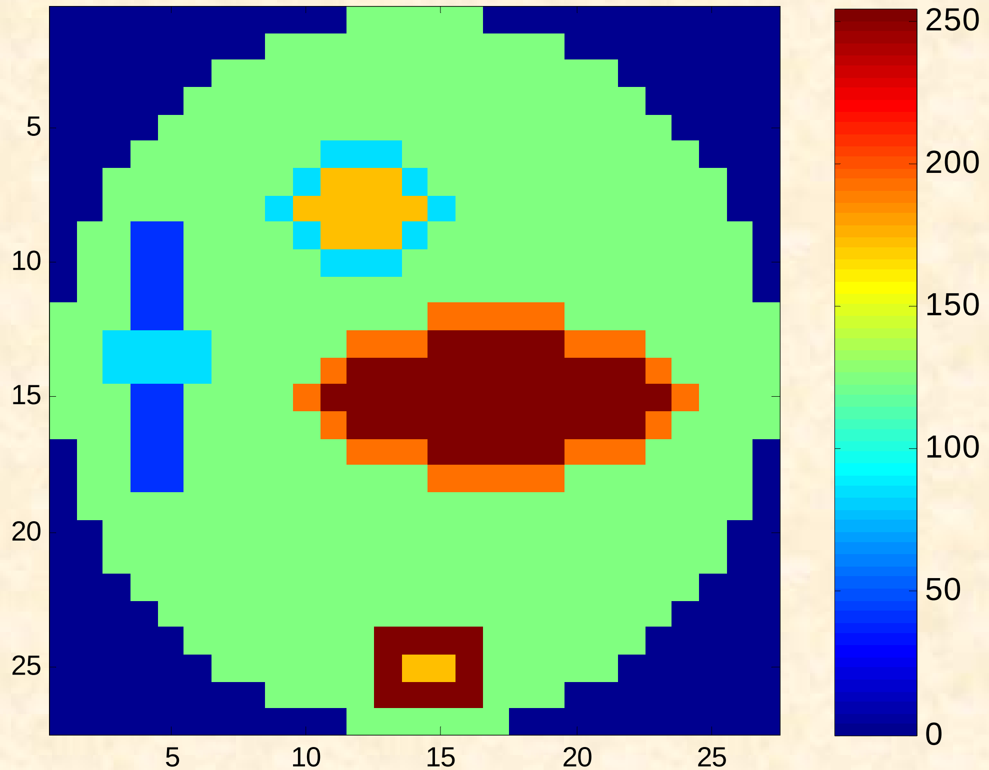
- Greatly reduces the number of free parameters in the imaging problem

SIMULATION RESULTS – LAND-USE AGGREGATION

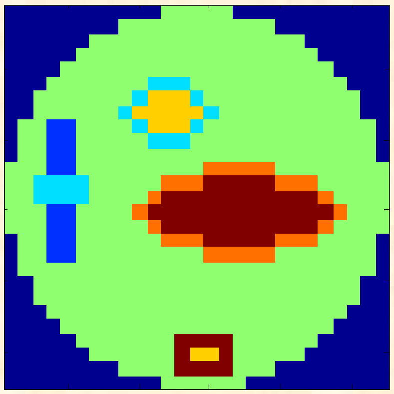
True Scattering Function



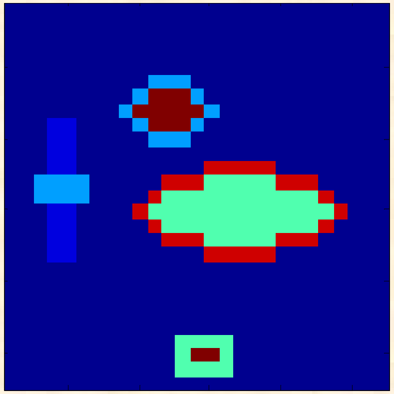
Estimated Scattering Function



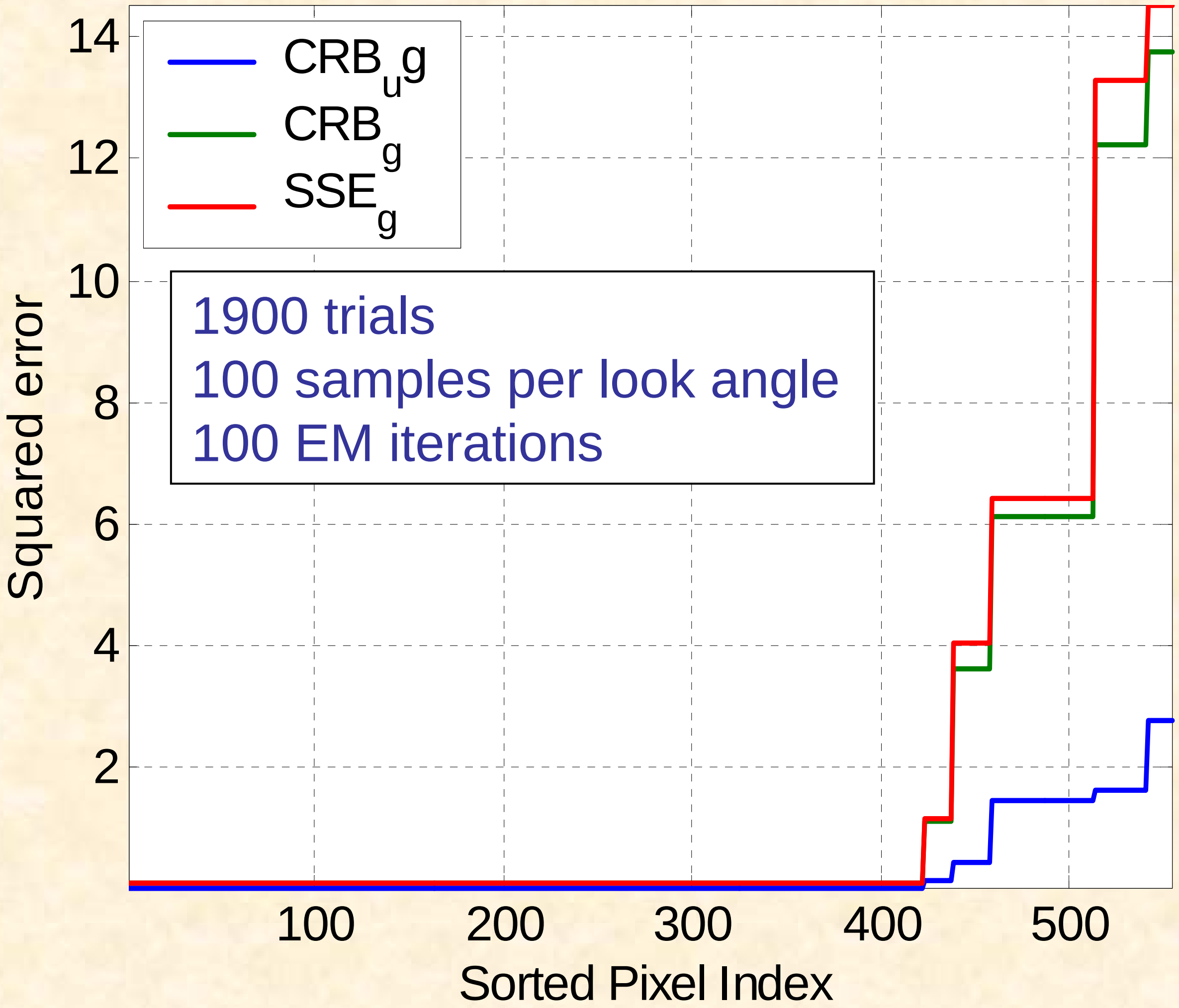
CRAMER-RAO BOUND WITH LAND-USE AGGREGATION
(sorted in ascending order of CRB)



Sample mean



Squared error



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MOVING TARGET DETECTION

- Adaptive Matched Filter (AMF) test statistic:

$$t(n) = \max_{v} \frac{\left| \mathbf{a}^{H}(n,v)\, \mathbf{R}^{-1}\, \mathbf{z} \right|^{2}}{\mathbf{a}^{H}(n,v)\, \mathbf{R}^{-1} \mathbf{a}(n,v)}$$

where:

$$n \in [1, N]$$

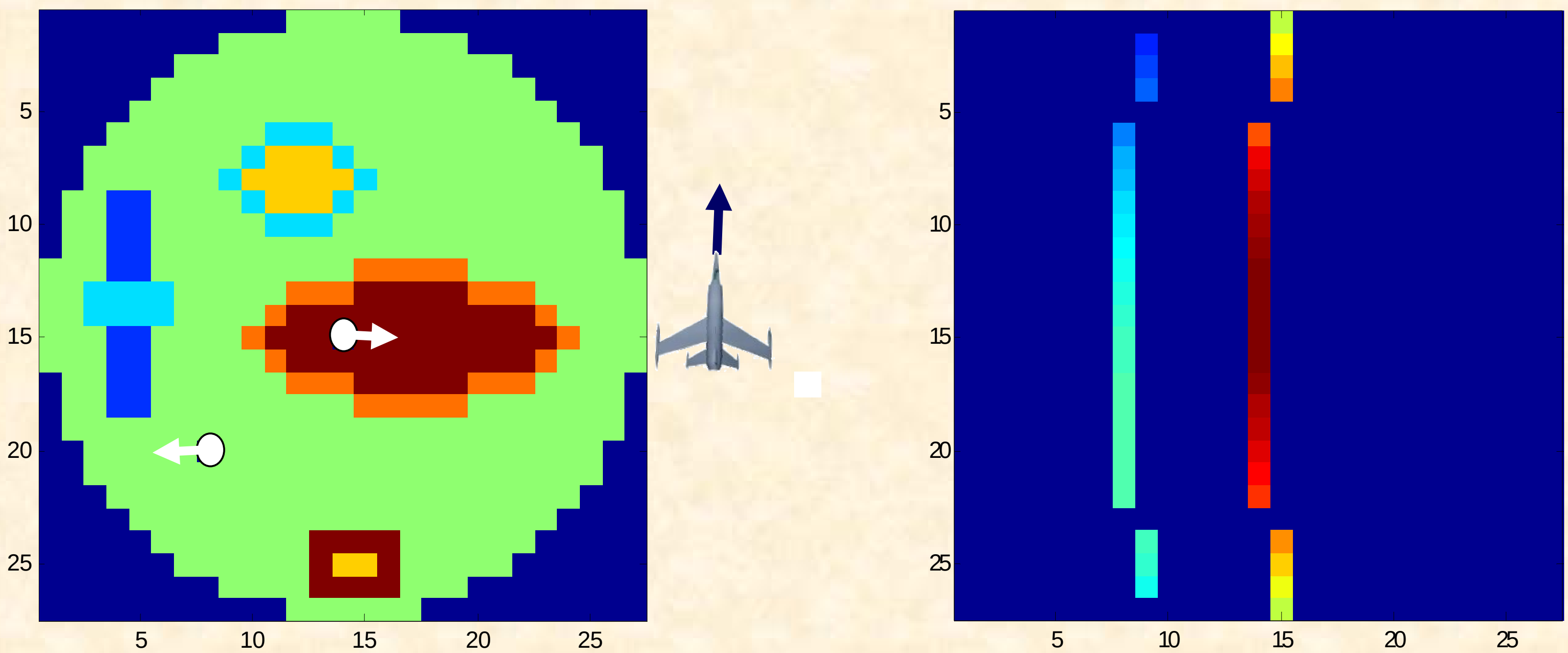
$$\mathbf{a}(n,v)_{\mathrm{MDx1}} = \mathbf{a}_{s}(\theta)_{\mathrm{Mx1}} \ \mathrm{x}\ \mathbf{a}_{D}(v)_{\mathrm{Dx1}}$$

$$\mathbf{R} = \mathbf{A}\, \hat{\boldsymbol{\Sigma}}\, \mathbf{A}^{H}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}(1,0) & \mathbf{a}(2,0) & \ldots & \mathbf{a}(N,0) \end{bmatrix}_{\mathrm{MDxN}}$$

MOVING TARGET DETECTION

Computed test statistic



Note: because the target velocity is unknown, target localization in angle is only as good as the spatial resolution of the radar array.

OUTLINE

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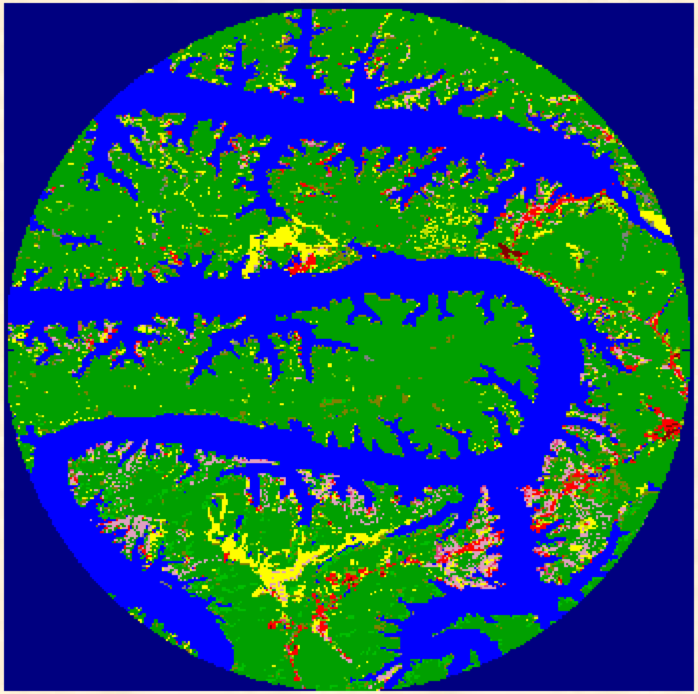
FULL-SCALE SIMULATION

- **Region of Interest**
  - Lake of the Ozarks
  - 15 km diameter
  - 197,316 pixels
  - 30m resolution



DATASETS

- Obtained from USGS Seamless Data Server
  - 30m resolution
- Digital Elevation Model
  - Used for modeling geometry
- Land Use
  - Scattering function based on 21 classes of land cover
    - 9 primary classes
      - Water, Developed, Barren, Forested Upland, Shrubland, Non-Natural Woody, Herbaceous Upland Natural/Semi-natural Vegetation, Herbaceous Planted/Cultivated, Wetlands
    - Each class contains one or more categories, e.g.
      - Open Water, High-Intensity Residential, Deciduous Forest, Row Crops
  - Scattering function chosen arbitrarily for simulation

SIMULATION PARAMETERS

- Platform
  - Flies in circular path around region
  - Radius 25 km
  - Altitude 7 km
  - 8 different viewpoints

- Radar
  - f_c : 10 GHz
  - BW: 10 MHz
  - PRF: 2 KHz
  - Pulses per CPI: 38
  - ULA elements: 12
  - Range gates: 990

DATACUBE GENERATION

- Response of a single patch

$$z_n = a(n,k)\sigma_n\lambda_{nk}$$

Incident energy on n^{th} patch on k^{th} pulse

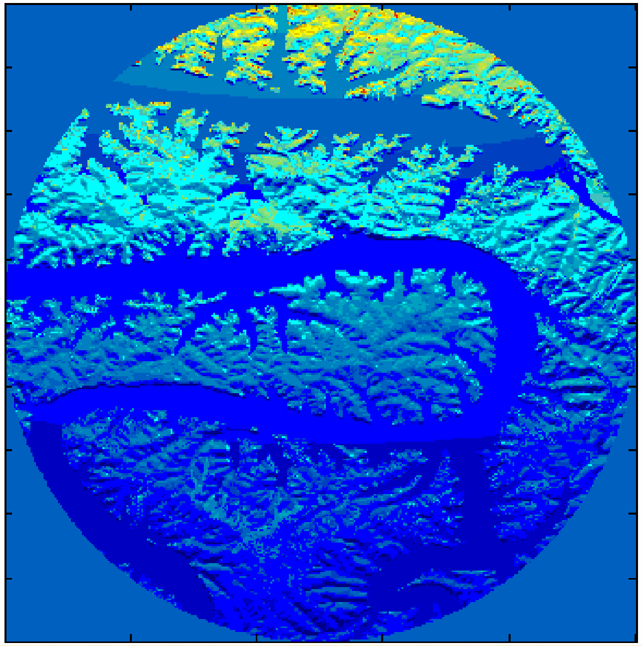
Scattering function at n^{th} patch

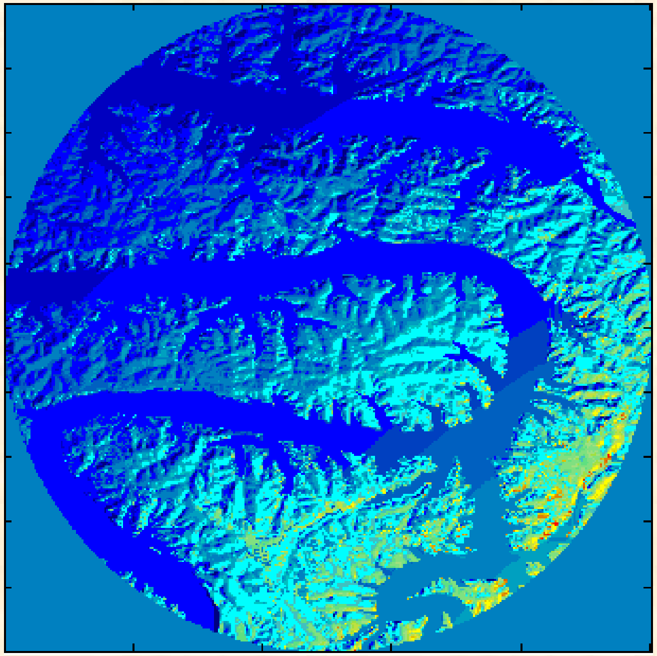
Kronecker product of spatial and Doppler response vectors

- Incident energy incorporates range and projected area of patch
- Patches hidden from radar are removed using Z-Buffer algorithm
  - Patches sorted by distance from radar
  - Any patch facing backwards or directly behind another is removed
- Response at a single range gate
  - Sum over all patches in range gate

ILLUMINATION

Illumination from different looks





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CONCLUSIONS

- Presented problem of radar imaging from multiple viewpoints and multiple noncoherent data sets as a maximum-likelihood structured covariance estimation problem
- Derived and implemented EM Algorithm
- Low-dimensional simulation results consistent with Cramér - Rao bound
- Land-use aggregation greatly reduces estimation error
- Resulting covariance estimates may be used for adaptive detection
- Full-scale simulation effort underway